value is $L_m = 0.5253$. From Eq. (39), the D_j's are 6.571, 5.562, and 4.097, respectively, and the values for $\beta_j = (D_j - 1)/R_j$ are 0.2933, 0.2401, and 0.3441, respectively. From Eq. (35), $T_1 = 76.64$ sec, $T_2 = 77.88$ sec, and $T_3 = 54.04$ sec. From Eq. (33) $v_1 = 8278$ fps, $v_2 = 9847$, and $v_3 = 6875$. Of course, the method presented here, unlike the method of Schurmann,[3] provides for cases in which the I_j's differ.

Solution Neglecting Drag and Gravity

When gravity is neglected for all stages, $d_j = 0$ and Eq. (34) becomes

$$D_j = D_m/[D_m - (D_m - 1)C_m/C_j] \tag{40}$$

The u_j's are given by Eq. (37) where $d_j = 0$, and U by Eq. (36). Figure 2 can be used to solve all problems of this class. One simply has to find a D_m in Fig. 2 such that the u_j's sum to U.

Example problem 3

Consider a 4-stage rocket with values for C_j/g_0 of 250, 275, 300, and 325 sec, respectively, with λ_j's of 0.9, 0.85, 0.8, and 0.75, respectively, and with $V_t = 40{,}000$ fps. We choose $m = 1$, and the values for C_j/C_m become 1, 1.1, 1.2, and 1.3, respectively, and U is determined to be 3.14 from Eq. (36). A first estimate for D_m of 2.32 is obtained from Fig. 2 at the point corresponding to the average u_j (i.e., $U/4$) and the average C_j/C_m. The correct solution for D_m in Fig. 2 with u_j's which sum to $U = 3.14$ is 2.31. The values for D_j are now determined from Eq. (40) and from $\bar{\beta}_j = \beta_j = (D_j - 1)/R_j$, the β_j's are 0.1455, 0.189, 0.224, and 0.258, respectively. The v_j's are determined from Eq. (33) with $\bar{g}_j = 0$, and $p_n/W_{1_i} = 1/630$.

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A Schlieren Technique for Measuring Jet Penetration into a Supersonic Stream

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Introduction

JET penetration into a supersonic stream may be determined from concentration measurements (Refs. 1–5), or more readily from schlieren photographs (Refs. 4 and 6).

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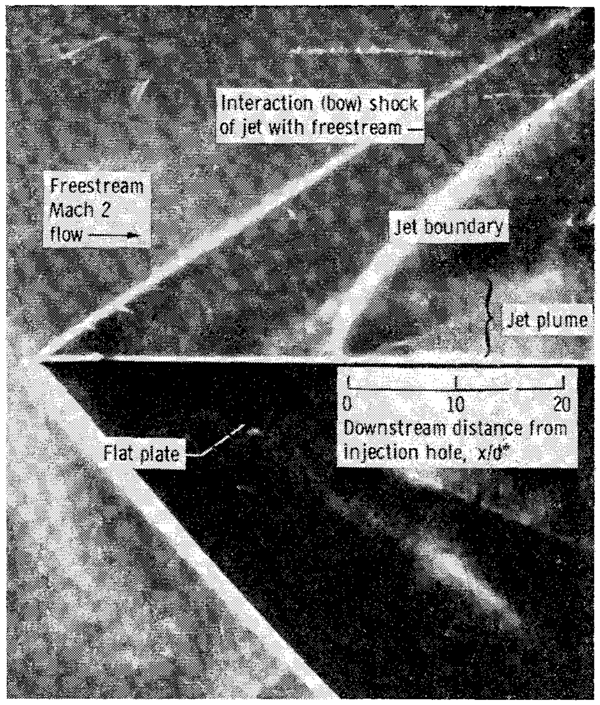
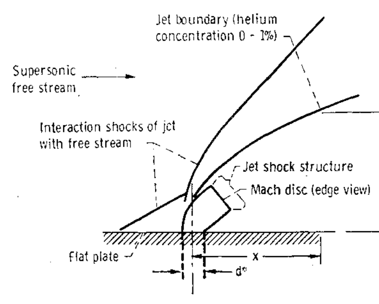


Fig. 1 Typical schlieren photograph (negative). Injection Mach number, 1.0; injection pressure, 100 psia (6.89×10^5 N/m²).

The essential features of the jet flowfield, as seen on a schlieren photograph, are diagramed in Fig. 1. Schetz, Hawkins, and Lehman[6] used the location of the center of the Mach disk as a measure of jet penetration. Zukoski and Spaid[4] used the upper portion of the shock structure as a criterion of penetration. The jet shock structure is located close to the injection orifice. Therefore, penetration measurements based on location of the jet shock structure may not completely describe the downstream behavior of the jet. Furthermore, it appears that for some injectants or operating conditions, the jet shock structure is difficult to detect, or may even be completely invisible. An alternate optical method for measuring jet penetration is thus desirable. In this study it is noted that the path of a helium jet may be detected by schlieren photography. The jet appeared as a plume shaped streak in the flowfield, as shown in Fig. 1. Measurements of helium jet penetration into a Mach 2 airstream were made by densitometer analysis and visual inspection of schlieren photographs and compared to results based on concentration measurements.

Helium was injected from a flat plate into a Mach 2 airstream. The injection was normal to the freestream, and injection Mach numbers were 1, 2.4, 3.5, and 4. The injection total pressure was varied from 48 psia (3.30×10^5 N/M²) to 130 psia (8.96×10^5 N/M²). The injection throat diameter for all injection conditions was 1.9 mm. Freestream total pressure and temperature were 0.92 atm (9.3×10^4 N/M²), and 625°R (347°K).

Fig. 2 Jet flowfield.



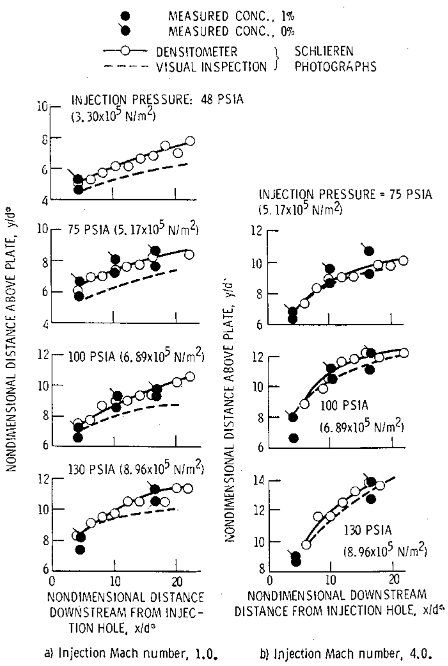


Fig. 3 Helium jet boundary, comparison of measurement techniques.

Centerplane concentration measurements obtained with a mass spectrometer were made at 4, 10, and 17 injection throat diameters downstream of the injection point. Details of the concentration measurement technique are given in Ref. 2.

Schlieren photographs of 0.1 sec exposure were made with a conventional single pass system with the knife edge oriented perpendicular to the freestream flow. The photographs were analyzed with a double beam recording microdensitometer. The densitometer beam scanned the photographs in a direction normal to the flat plate 4 to 20 injection throat diameters downstream of the injection point. The distance from the surface of the flat plate to the point where the photographic density decayed to the background level was considered to be the jet penetration, or boundary location, at each downstream position. The apparent outer edge of the jet, that appeared as a longitudinal streak, was also measured by visual inspection of the photographs.

Results and Discussion

Typical results of jet penetration obtained from densitometer measurements, visual inspection of the schlieren photographs, and concentration measurements are shown in Fig. 3. The penetration distance normal to the flat plate y and the downstream distance from the injection orifice x are nondimensionalized by the injection throat diameter, d^*. The solid points represent volumetric helium concentrations of 0 and 1% on the centerplane.

In most instances the jet boundary obtained by densitometer analysis of the schlieren photographs was found to lie within the region bounded by the 0 and 1% concentration points. It thus appears, that if the jet can be visualized by the schlieren method, penetration can be quite accurately measured by the technique described in this Note. The visually located boundary, indicated as the "jet boundary" in Fig. 1, in some cases corresponded to the region bounded by the 0 and 1% concentration points. However, use of the visually located boundary underestimated penetration by as much as 15%. Visual measurements might however be acceptable for rapid qualitative estimates of penetration. Independent measurements of the jet boundary location from the schlieren negatives by two observers were identical. It was also established that effects of photographic emulsion type, exposure time, and schlieren sensitivity settings had little effect on the accuracy of the jet boundary measurements. This was established by making measurements using Panatomic and Tri-X film types, varying the exposure time from 0.01 to 0.005 sec, and varying the adjustment of the knife edge.

Zukoski and Spaid,[4] observed what they considered to be a jet streamline with nitrogen and argon injection. They could not however observe this feature with helium injection. This feature was most likely, as suggested by Schetz, Hawkins, and Lehman,[6] a portion of the jet shock structure. In the present study it was also noted that jet shock structure could be observed with nitrogen and argon but not with helium injection. On the other hand, the streak indicating the injectant path, as shown on Fig. 1 was observed only for helium. Thus, the technique described here, may be useful only for helium injection. However, the technique described in this Note may provide a valuable alternate method of measuring jet penetration for gases such as helium, where the injectant path rather than the shock structure can be visualized.

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Laser Velocimeter Used to Find Transition Region

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OSBORNE Reynolds[1] is credited with finding the unifying principle for explaining the phenomena of the transition of flow from the laminar regime to the turbulent regime first observed and investigated by Hagen.[2,3] Reynolds measured the pressure drop through a known length of pipe, used the average velocity $\bar{V}$ to calculate $D\bar{V}\rho/\mu$ (the Reynolds number, Re), and determined the critical Reynolds number (Re_{crit}) for transition as the value at which the friction coefficient calculated from the pressure drop took a compara-

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